WE1E-5

Uncertainty Estimation and Optimal Extraction of Intrinsic FET Small Signal Model Parameters

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Abstract— In this paper, analytical expressions for the sensitivities in the parameters of a standard intrinsic FET small signal model are derived with respect to variations in the *S*-parameters. The sensitivities are used to estimate the uncertainty in extracted model parameters.

The theories are applied to measurements made on a commercial HEMT device. Using models for the measurement uncertainties allows the model parameter uncertainties to be studied versus frequency and bias. As a result, optimal, minimum uncertainty parameter extraction can be performed independent of the bias voltage and without prior knowledge of the FET device characteristics. Thus making it suitable for implementation in automatic multi-bias extraction programs.

Index Terms—FET, model, small signal, sensitivity, parameter extraction, error estimation, measurement uncertainty

I. Introduction

For MESFETs and HEMTs, the procedure of extracting small signal model parameters is well established. Usually direct extraction methods are used [1-4].

During the extraction process the measurement uncertainties will give rise to a corresponding uncertainty in the model parameters. Knowledge about the parameter uncertainties would allow calculation of their influence on circuit performance using e.g. sensitivity analysis [5, 6]. However, little work has been reported on how to find the uncertainty with which the model parameters can be extracted.

In [7] King *et al.* give quantitative figures for the uncertainty of intrinsic FET model parameters. Analytical expressions have been used, but since only numerical figures are given at a single frequency and bias point, it is difficult to draw any general conclusions from those results. Walters *et al.* present in [8] experimental results for measurement uncertainties and the resulting uncertainty in extracted small signal parameters. No details are given about the model parameter uncertainty calculations performed.

In this paper, analytical relations between the intrinsic model parameter- and measured *S*-parameter uncertainties are presented. The model parameter uncertainties can be used to perform optimal parameter extractions at each bias point of interest without having prior knowledge about the device characteristics.

The parameter uncertainties obtained can furthermore be used in sensitivity calculations for evaluation of the uncertainty in circuit responses and other purposes.

II. Sensitivities and Uncertainty Estimation

The relative sensitivity[1], K, in a parameter x for relative changes in e.g. S_{11} is defined as

$$K_{S_{11}}^{x} \triangleq \frac{\frac{\partial x}{x}}{\frac{\partial S_{11}}{S_{11}}} = \frac{\partial x}{\partial S_{11}} \frac{S_{11}}{x} \tag{1}$$

If x only depends on S_{11}, the relative change in x can be related to the relative change in S_{11} using the sensitivity in (1):

$$\frac{\Delta x}{x} \cong K_{S_{11}}^{x} \frac{\Delta S_{11}}{S_{11}} \tag{2}$$

In a FET modeling context the sensitivities can typically be used to calculate a relative change in the transconductance for a relative change in the measured magnitude of S_{21}.

Usually, the model parameters depend on all measured *S*-parameters. Thus, when calculating the change in x, the contributions from all *S*-parameters must be considered and added. The change in x can then be expressed in a compact form as

$$\frac{\Delta x}{x} \cong \sum_{\forall k,l \in \{1,2\}} K_{S_{kl}}^{x} \frac{\Delta S_{kl}}{S_{kl}} \tag{3}$$

In fact, this represents a first order Taylor series expansion of x in terms of the complex *S*-parameters. It is therefore valid only as long as the ΔS_{kl} are sufficiently small. For larger ΔS_{kl}, higher order derivatives/sensitivities must be taken into account. However, for most parameters, the first order approximation is good enough considering the relatively high accuracy obtained in *S*-parameter measurements. Hereafter, the approximation in (3) is considered valid with equal sign used.

The measured *S*-parameters are complex variables. Thus, uncertainties may be expressed in terms of magnitude and phase deviations. Since, in the sensitivity calculations, every contribution must be added the deviation in x will be expressed in the measured *S*-parameters as

[1] The notation K is used instead of S for the relative sensitivities to avoid confusion with the *S*-parameters.

TABLE I
EXTRINSIC Y-PARAMETER SENSITIVITIES

$K_S^{Y_e}$	S_{11}	S_{12}	S_{21}	S_{22}
$Y_{e,11}$	$\frac{2S_{11}(1+S_{22})^2}{\Delta_1\Delta_3}$	$-\frac{2S_{12}S_{21}(1+S_{22})}{\Delta_1\Delta_3}$	$-\frac{2S_{12}S_{21}(1+S_{22})}{\Delta_1\Delta_3}$	$\frac{2S_{12}S_{21}S_{22}}{\Delta_1\Delta_3}$
$Y_{e,12}$	$-\frac{S_{11}(1+S_{22})}{\Delta_3}$	$\frac{(1+S_{11})(1+S_{22})}{\Delta_3}$	$\frac{2S_{12}S_{21}}{\Delta_3}$	$-\frac{(1+S_{11})S_{22}}{\Delta_3}$
$Y_{e,21}$	$-\frac{S_{11}(1+S_{22})}{\Delta_3}$	$\frac{2S_{12}S_{21}}{\Delta_3}$	$\frac{(1+S_{11})(1+S_{22})}{\Delta_3}$	$-\frac{(1+S_{11})S_{22}}{\Delta_3}$
$Y_{e,22}$	$\frac{2S_{11}S_{12}S_{21}}{\Delta_2\Delta_3}$	$-\frac{2(1+S_{11})S_{12}S_{21}}{\Delta_2\Delta_3}$	$-\frac{2(1+S_{11})S_{12}S_{21}}{\Delta_2\Delta_3}$	$\frac{2(1+S_{11})^2 S_{22}}{\Delta_2\Delta_3}$

$$\frac{\Delta x}{x} = \sum_{\forall k,l\in\{1,2\}} K^x_{|S_{kl}|}\frac{\Delta|S_{kl}|}{|S_{kl}|} + K^x_{\angle S_{kl}}\Delta\angle S_{kl} \quad (4)$$

where it has been used that the magnitude and phase deviations are usually specified in relative and absolute terms respectively. It can be shown that the real magnitude and phase sensitivities in (4) are related to the same complex sensitivities in (3) with the absolute phase deviations given in radians,

$$K^x_{|S_{kl}|} = K^x_{S_{kl}} \quad (5)$$

$$K^x_{\angle S_{kl}} = jK^x_{S_{kl}} \quad (6)$$

It is therefore only necessary to find the sensitivities with respect to complex S-parameter variations in the following calculations.

The measurement uncertainties are characterized by their statistical properties. If the S-parameter deviations are assumed to be normal distributed having a zero mean and being uncorrelated makes it possible to use (4) to estimate the variance in x in terms of the relative magnitude variance, $\sigma^2_{|S_{kl}|}$, and the absolute phase variance, $\sigma^2_{\angle S_{kl}}$,

$$\sigma_x^2 = \sum_{\forall k,l\in\{1,2\}} \left(K^x_{|S_{kl}|}\right)^2 \sigma^2_{|S_{kl}|} + \left(K^x_{\angle S_{kl}}\right)^2 \sigma^2_{\angle S_{kl}} \quad (7)$$

III. PARAMETER SENSITIVITY CALCULATIONS

The uncertainties in the measured S-parameters will propagate to the model parameters in the same way as the model parameters are extracted. Hence, the sensitivity analysis may be carried out in parallel with the small signal parameter extraction to find their uncertainties.

The first step is to convert the S-parameters into extrinsic Y-parameters, e.g. by [9],

$$Y_e = -\frac{1}{\Delta_3}\begin{bmatrix}\Delta_1 & 2S_{12}\\ 2S_{21} & \Delta_2\end{bmatrix} \quad (8)$$

where

$$\Delta_1 = (S_{11}-1)(S_{22}+1) - S_{12}S_{21} \quad (9)$$

$$\Delta_2 = (S_{11}+1)(S_{22}-1) - S_{12}S_{21} \quad (10)$$

$$\Delta_3 = (S_{11}+1)(S_{22}+1) - S_{12}S_{21} \quad (11)$$

It is then straightforward to derive all extrinsic Y-parameter sensitivities using the definition in (1). The resulting sensitivities are collected in Table I.

If the parasitic element values are known, e.g. from a cold-FET extraction, the intrinsic Y-parameters can be found using de-embedding techniques [1, 10]. The intrinsic Y-parameter sensitivities can then be derived from the extrinsic Y-parameter sensitivities. Our experience is, however, that the resulting differences between the extrinsic- and intrinsic sensitivities are small—especially if the influence of the device parasitic elements is small, such as in on-wafer measurements.

When the intrinsic Y-parameters are found, the model parameters are usually calculated analytically. Fig. 1 shows a commonly used intrinsic small signal model shown to be valid up to very high frequencies [3].

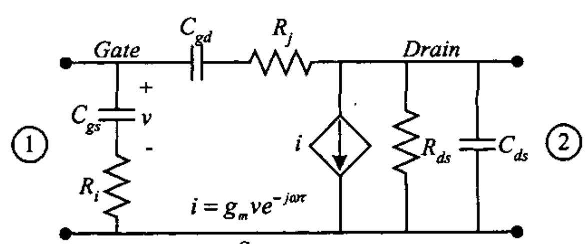

Fig. 1. High frequency intrinsic FET small signal model.

The derivation of the model parameter sensitivities will be shown only for C_{gs} and R_i but is similar for the other parameters.

C_{gs} and R_i can be determined from the intrinsic admittance parameter Y_{gs},

$$Y_{gs} = Y_{i,11} + Y_{i,12} = \frac{j\omega C_{gs}}{1 + j\omega C_{gs} R_i} \quad (12)$$

The relative sensitivity in Y_{gs} can then be related to the C_{gs} and R_i sensitivities by

$$K^{Y_{gs}}_{S_{kl}} = \frac{\partial Y_{gs}}{\partial R_i} K^{R_i}_{S_{kl}} \frac{R_i}{Y_{gs}} + \frac{\partial Y_{gs}}{\partial C_{gs}} K^{C_{gs}}_{S_{kl}} \frac{C_{gs}}{Y_{gs}} \quad (13)$$

Evaluating the partial derivatives yields after some simplifications

$$\mathrm{K}_{S_{kl}}^{Y_{gs}} = -\mathrm{K}_{S_{kl}}^{R_i} Y_{gs} R_i + Y_{gs} \mathrm{K}_{S_{kl}}^{C_{gs}} / j\omega C_{gs} \quad (14)$$

Since the sensitivities in the real model parameters, C_{gs} and R_i, also must be real, their sensitivities can be identified from the real and imaginary parts of (14):

$$\mathrm{K}_{S_{kl}}^{R_i} = \mathrm{Re}\left(-\mathrm{K}_{S_{kl}}^{Y_{gs}} / Y_{gs}\right) / R_i \quad (15)$$

$$\mathrm{K}_{S_{kl}}^{C_{gs}} = \omega C_{gs} \,\mathrm{Im}\left(-\mathrm{K}_{S_{kl}}^{Y_{gs}} / Y_{gs}\right) \quad (16)$$

The complete list of parameter sensitivities with respect to magnitude deviations is collected in Table II. The absolute phase sensitivities are, as discussed in the previous section, found by multiplying the complex sensitivities in Table II by the imaginary unit.

TABLE II
INTRINSIC MODEL PARAMETER SENSITIVITIES

Parameter, x	Relative magnitude sensitivity, $\mathrm{K}_{\lvert S_{kl} \rvert}^{x}$
R_i	$\mathrm{Re}\left(-\mathrm{K}_{S_{kl}}^{Y_{gs}} / Y_{gs}\right) / R_i$
C_{gs}	$\omega C_{gs} \,\mathrm{Im}\left(-\mathrm{K}_{S_{kl}}^{Y_{gs}} / Y_{gs}\right)$
R_j	$\mathrm{Re}\left(-\mathrm{K}_{S_{kl}}^{Y_{gd}} / Y_{gd}\right) / R_j$
C_{gd}	$\omega C_{gd} \,\mathrm{Im}\left(-\mathrm{K}_{S_{kl}}^{Y_{gd}} / Y_{gd}\right)$
g_{ds}	$\mathrm{Re}\left(Y_{ds} \mathrm{K}_{S_{kl}}^{Y_{ds}}\right) / g_{ds}$
C_{ds}	$\mathrm{Im}\left(Y_{ds} \mathrm{K}_{S_{kl}}^{Y_{ds}}\right) / \omega C_{ds}$
g_m	$\mathrm{Re}\left(\mathrm{K}_{S_{kl}}^{Y_m} + R_i Y_{gs} \left(\mathrm{K}_{\lvert S_{kl} \rvert}^{R_i} + \mathrm{K}_{\lvert S_{kl} \rvert}^{C_{gs}}\right)\right)$
τ	$-\mathrm{Im}\left(\mathrm{K}_{S_{kl}}^{Y_m} + R_i Y_{gs} \left(\mathrm{K}_{\lvert S_{kl} \rvert}^{R_i} + \mathrm{K}_{\lvert S_{kl} \rvert}^{C_{gs}}\right)\right) / \omega\tau$

The admittance parameter sensitivities are given by

$$\mathrm{K}_{S_{kl}}^{Y_{gs}} = \mathrm{K}_{S_{kl}}^{Y_{i,11}} \, Y_{i,11} / Y_{gs} + \mathrm{K}_{S_{kl}}^{Y_{i,12}} \, Y_{i,12} / Y_{gs} \quad (17)$$

$$\mathrm{K}_{S_{kl}}^{Y_{gd}} = \mathrm{K}_{S_{kl}}^{Y_{i,12}} \quad (18)$$

$$\mathrm{K}_{S_{kl}}^{Y_m} = \mathrm{K}_{S_{kl}}^{Y_{i,21}} \, Y_{i,21} / Y_m - \mathrm{K}_{S_{kl}}^{Y_{i,12}} \, Y_{i,12} / Y_m \quad (19)$$

$$\mathrm{K}_{S_{kl}}^{Y_{ds}} = \mathrm{K}_{S_{kl}}^{Y_{i,12}} \, Y_{i,12} / Y_{ds} + \mathrm{K}_{S_{kl}}^{Y_{i,22}} \, Y_{i,22} / Y_{ds} \quad (20)$$

Once the parameter sensitivities are known, their uncertainties can be estimated in terms of the S-parameter uncertainties using (7).

IV. EXPERIMENTAL RESULTS

Measurements on a HEMT device are used together with a measurement uncertainty model to estimate the uncertainty in the extracted parameter values. The transistor used is made in OMMIC's D01PH GaAs process [11], and was measured with a 50 GHz Agilent 8510C vector network analyzer (VNA) using coplanar probing techniques. Measurements made in the saturated region, used for maximum gain are used to demonstrate the uncertainty calculations.

The parasitic elements were initially determined with the cold-FET method [1, 3]. Two of the intrinsic model parameters, g_m and R_j, were studied, where g_m is normally easy to extract whereas R_j has a significant contribution only at higher frequencies and therefore more difficult to extract. The frequency dependence of these parameters is shown in Fig. 2.

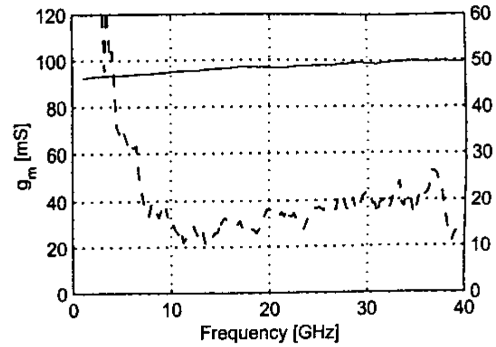

Fig. 2. Extracted g_m (solid) and R_j (dashed) versus frequency.

As apparent from Fig. 2, the relative variations in R_j are much larger than in g_m making it difficult to decide which value is the better to use.

To perform the parameter uncertainty calculations versus frequency, the S-parameter uncertainties must also be known. For this purpose an S-parameter uncertainty model has been developed for the relative magnitude and absolute phase uncertainties from the VNA specifications [12]. Evaluation of the measurement accuracy using verification standards has shown that the uncertainties achieved using a careful on-wafer TRL calibration [13] is close to the one specified. The model and specification is shown for S_{11} and S_{22} in Fig. 3. Note that the relative magnitude and absolute phase uncertainty specifications are coincident.

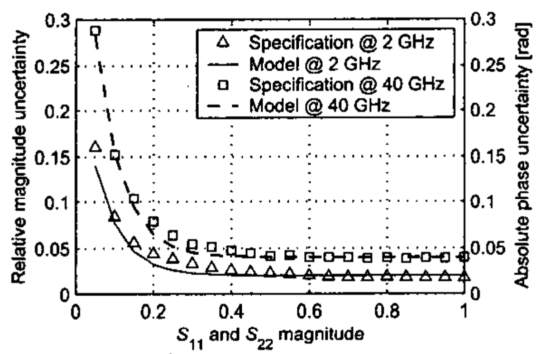

Fig. 3. Specified and modeled uncertainty in S_{11} and S_{22}.

Using the uncertainty models together with the sensitivity expressions in (7) makes it possible to evaluate the uncertainty in the extracted model parameters for each measurement frequency and bias point of interest. Fig. 4 shows the calculated uncertainty for g_m, R_j, and C_{gs} versus frequency in the same bias point as used before.

Fig. 4 shows that g_m should be extracted at a low frequency while R_j should be extracted at high frequency where the parameter uncertainty is minimal. C_{gs} has an intermediate optimal extraction frequency where the uncertainty is minimal. Table III shows all extracted parameters with corresponding estimated uncertainties and their optimal extraction frequencies, f_{opt}.

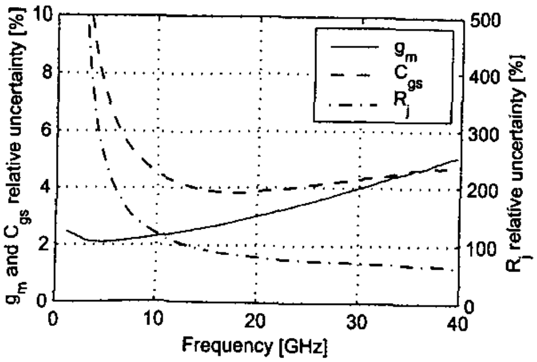

Fig. 4. Estimated relative uncertainty in g_m, C_{gs}, and R_j versus frequency.

TABLE III
EXTRACTED MODEL PARAMETERS WITH UNCERTAINTY ESTIMATION

Parameter	Value	σ_x [%]	f_{opt} [GHz]
R_i	0.4 [Ω]	340	40
C_{gs}	136 [fF]	3.8	18
R_j	14 [Ω]	62	40
C_{gd}	14.8 [fF]	1.4	6.3
g_{ds}	4.80 [mS]	4.2	1.0
C_{ds}	26 [fF]	10	29
g_m	93.6 [mS]	2.1	4.5
τ	0.38 [ps]	39	38

As can be seen in Table III, the uncertainty in R_i prevents extraction of its value with any confidence.

Usually, when automatic extractions are performed for multiple bias points, constant frequency ranges are set for each model parameter over which the parameters are averaged [14]. Using the presented procedure, optimal values can be extracted automatically for every bias point, without any prior knowledge of the frequency dependence of the model parameters or the device behavior.

V. CONCLUSIONS

A derivation of theoretical relations between the uncertainties in measured S-parameters and the resulting uncertainties in the intrinsic model parameters have been presented. Instrument specifications have been used to model the S-parameter uncertainties. This results in a systematic way of extracting the intrinsic model parameters with minimal uncertainty. Since no assumptions about the device characteristics are made, we believe that it is well suited for implementation in automatic multi-bias extraction programs. The uncertainties in the extracted parameters can be used to track the model accuracy during the extractions, to calculate the uncertainty in circuit responses, or to check the statistical significance of parameter variations in FET databases [15].

ACKNOWLEDGMENTS

The authors would like to thank Dr. Mikael Garcia, Dr. Klas Yhland, Mr. Kristoffer Andersson, and Mr. Lars Landén for fruitful discussions and the Swedish Foundation for Strategic Research for financial support.

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